Benha University | **Antennas &Wave Propagation** | **Electrical Eng. Dept.**
Faculty of Engineering | **4th year communication**
Shoubra | **2013-2014**

Sheet (6) - solution

1. Design a four –element ordinary end fire array with the elements placed along the Z-axis a distance d apart with the maximum of the array factor directed toward θ=0°. for a spacing of d=λ/2 between the elements find the

(a) Progressive phase excitation between the elements to accomplish this.

(b) Angles (in degrees) where the nulls of the array factor occur.

(c) Angles (in degrees) where the maximum of the array factor occur.

(d) Beamwidth (in degrees) between the first nulls of the array factor.

(e) Directivity (in dB) of an array factor.

a. $\beta = -kd = -\frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right) = -\pi = -180^\circ$

b. $\theta_n = \cos^{-1}\left[1-\frac{n\lambda}{Nd}\right] = \cos^{-1}\left(1-\frac{n\lambda}{4\lambda/2}\right) = \cos^{-1}\left(1-\frac{n}{2}\right)$, $n=1,2,3,\cdots, n\neq 4,8,\cdots$

$n=1: \theta_1 = \cos^{-1}(1/2) = 60^\circ$

$n=2: \theta_2 = \cos^{-1}(0) = 90^\circ$

$n=3: \theta_3 = \cos^{-1}(-1/2) = 120^\circ$

c. $\theta_m = \cos^{-1}(1-m\lambda/d) = \cos^{-1}(1-m\lambda/\lambda/2) = \cos^{-1}(1-2m)$, $m=0,1,2,\cdots$

$m=0: \theta_0 = \cos^{-1}(1) = 0^\circ$

$m=1: \theta_1 = \cos^{-1}(-1) = 180^\circ$

d. $\Theta_0 = 2\cos^{-1}\left(1-\frac{\lambda}{Nd}\right) = 2\cos^{-1}\left(1-\frac{\lambda}{4\lambda/2}\right) = 2\cos^{-1}\left(1-\frac{1}{2}\right) = 2\cos^{-1}\left(\frac{1}{2}\right) = 2(60^\circ)$

$\Theta_0 = 120^\circ$

e. $D_0 = 4N\left(\frac{d}{\lambda}\right) = 4(4)\left(\frac{\lambda/2}{\lambda}\right) = 8 = 9.03\,dB$

2. Arrays of 10 isotropic elements are placed along z-axis a distance d=λ/4 apart. Assuming uniform distribution. Find for both broadside and ordinary end-fire cases the following:

(a) Progressive phase (in degrees).

(b) First side lobe level beam width.

(c) Directivity (in dB).

$N=10, \quad d=\lambda/4$

a. Broadside (Table 6.1 and 6.2) $\Rightarrow \beta=0$

$$HPBW = 2\left[90^\circ - \cos^{-1}\left(\frac{1.391\times4}{10\pi}\right)\right] = 2(90^\circ - 79.80^\circ) = 20.4^\circ$$

$$FNBW = 2\left[90^\circ - \cos^{-1}\left(\frac{4}{10}\right)\right] = 2(90^\circ - 66.42^\circ) = 47.16^\circ$$

$$FSLBW = 2\left[90^\circ - \cos^{-1}\left(\frac{6}{10}\right)\right] = 2(90^\circ - 53.13^\circ) = 73.74^\circ$$

$$D_o = 2N\left(\frac{d}{\lambda}\right) = 2\cdot10\cdot\frac{1}{4} = 5 = 6.99\,dB$$

b. Ordinary End-Fire (Tables 6.3 and 6.4) $\Rightarrow \beta = \pm kd = \pm\pi/2 = \pm90^\circ$

$$HPBW = 2\cos^{-1}\left[1 - \frac{1.391(4)}{10\pi}\right] = 2(34.62^\circ) = 69.25^\circ$$

$$FNBW = 2\cos^{-1}\left[1 - \frac{4}{10}\right] = 2\cos^{-1}(0.6) = 2(53.13^\circ) = 106.26^\circ$$

$$FSLBW = 2\cos^{-1}\left[1 - \frac{3(4)}{10}\right] = 2(66.42) = 132.84^\circ$$

$$D_o = 4N\left(\frac{d}{\lambda}\right) = 4(10)\frac{1}{4} = 10 = 10\,dB$$

3. A uniform of 20 isotropic elements is placed along z-axis a distance $\lambda/4$ apart with a Progressive phase shift of "β". Calculate "β" (give the answer in radians) for the following array types:

(a) Broadside.

(b) End-fire with maximum at $\theta=0^\circ$.

(c) End-fire with maximum at $\theta=180^\circ$.

(d) Phased array with maximum aimed at $\theta=30^\circ$.

$$kd = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

a. $\beta = 0$ radians

b. $\beta = -\pi/2$

c. $\beta = +\pi/2$

d. $\beta = -1.36 = -\frac{\sqrt{3}}{4}\pi = -0.433\pi$

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4. Design a 19-element uniform linear scanning array with a spacing of λ/4 between the elements.

(a) What is the progressive phase excitation between the elements so that the maximum of the array factor is 30° from the line where the elements are placed?

(b) What is the HPBW in degrees of the array factor of part a.

$N=19, \; d=\lambda/4$

a. $\beta = -kd\cos\theta_0 \Big|_{\theta_0=30^\circ,\; d=\lambda/4} = -\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right)\cos(30^\circ) = -\frac{\pi}{2}\frac{\sqrt{3}}{2} = -\frac{\pi\sqrt{3}}{4} = -1.3603$

$\beta = -\frac{\pi\sqrt{3}}{4} = -1.3603 \text{ (rad)} = -77.942^\circ$

b. $\theta_h = \cos^{-1}\left[\cos\theta_0 - 0.443\frac{\lambda}{L+d}\right]_{\theta_0=30^\circ} - \cos^{-1}\left[\cos\theta_0 + 0.443\frac{\lambda}{L+d}\right]_{\theta_0=30^\circ}$

$= \cos^{-1}\left[0.866 - \frac{0.443}{5}\right] - \cos^{-1}\left[0.866 + \frac{0.443}{5}\right]$

$= \cos^{-1}(0.7774) - \cos^{-1}(0.9546) = 38.9769^\circ - 17.3309^\circ = 21.6459^\circ$

$\theta_h = 21.6459^\circ$

REPORT

1. The maximum distance d between the elements in a linear scanning array to suppress grating lobes is

$$d_{max} = \frac{\lambda}{1 + |COS(\theta o)|}$$

Dr. Gehan Sami

Where θo is the direction of the pattern maximum? What is the maximum distance between the elements, without introducing grating lobes, when the array is designed to scan to maximum angles of

(a) $\theta o = 30^{\circ}$.

(b) $\theta o = 45^{\circ}$.

(c) $\theta o = 60^{\circ}$.

The recommended element spacing is

$d = \frac{1}{1+\cos\theta}$, where θ is the scan angle in degrees

a. $\theta_o = 30^{\circ}$

$d = \frac{1}{1+\cos 30^{\circ}} = 0.5359$ wavelength

b. $\theta_o = 45^{\circ}$

$d = \frac{1}{1+\cos 45^{\circ}} = \frac{1}{1+0.7071} = 0.58578$ wave length

c. $\theta_o = 60^{\circ}$

$d = \frac{1}{1+\cos 60^{\circ}} = \frac{1}{1+0.5} = 0.6667$ Wavelength.

2. For a uniform broadside linear array of 10 isotropic elements, determine the approximate directivity (in dB) when the spacing between the element is

(a) λ/4 (b) λ/2 (c) 3 λ/4 (d) λ.

$D_o \simeq 2N(d/\lambda)$

a. $d = \frac{\lambda}{4}$, $D_o \simeq 2 \cdot 10 \cdot \frac{1}{4} = 5 = 6.99\text{dB}$

b. $d = \frac{\lambda}{2}$, $D_o \simeq 2 \cdot 10 \cdot \frac{1}{2} = 10 = 10\text{dB}$

c. $d = \frac{3\lambda}{4}$, $D_o \simeq 2 \cdot 10 \cdot (0.75) = 15 = 11.76\text{dB}$

d. $d = \lambda$, $D_o \simeq 2 \cdot 10 \cdot (1) = 20 = 13.0\text{dB}$

Good Luck

Dr. Gehan Sami